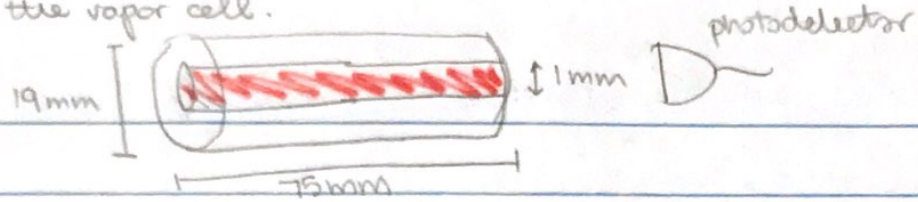
Let's say there is a beam of constant diameter 1mm passing the vapor cell.



Power measured by photodetector at a given time is, say, $8\ mW = 8\ \frac{mJ}{s}$

Assuming all photons have $\lambda = 780\ nm$, energy from each photon is $E = hc/\lambda = \frac{6.626 \cdot 10^{-34}\ Js \cdot 2.998 \cdot 10^{8}\ \frac{m}{s}}{780\ nm} = 2.54 \cdot 10^{-19}\ J$. So, the number of photons hitting the photodetector each second is:

$$\left(\frac{8\ mJ}{s}\right) / 2.54 \cdot 10^{-19} \frac{J}{photon} = 3.15 \cdot 10^{16}\ photons/s$$

↑ power (assuming beam size is smaller than active area).

The time it takes for a photon to travel the length of the vapor cell is (assume $c \sim 2.998 \cdot 10^{8}\ m/s$ even though Rb vapor has some index of refraction) $= \frac{75\ mm}{c} = 2.5 \cdot 10^{-10}\ s$.

So at any given time in the vapor cell (photons are in there for $2.5 \cdot 10^{-10}\ s$), there are $3.15 \cdot 10^{16} \frac{photons}{s} \cdot 2.5 \cdot 10^{-10}\ s$

$= \underline{7.88 \cdot 10^{6}\ photons}$.

Now how many atoms are in the vapor cell?
We aren't operating at insanely low T or high P/ρ (could check the critical temp/pressure of Rb vapor to confirm).

so let's use ideal gas law.

$$PV = nRT$$

- P: pressure of vapor
- V: volume of vapor cell (cylinder)
- n: # moles of Rb
- R: ideal gas constant $= 8.314 \frac{J}{K\,mol}$
- T: temp of vapor cell heater ~ 25°C say (= 298 K)

If we solve for n, we can use Avogadros # to get # of atoms (total) (in vapor cell)

$$\left(\# \text{ atoms} = n \cdot \frac{6.022 \cdot 10^{23} \text{ atoms}}{mol} \right)$$

$$V \sim 75\,mm \cdot \pi \left(19/2\ mm \right)^2 = 2.1 \cdot 10^{-5}\ m^3$$

The pressure exerted by the Rb vapor can be estimated with the vapor pressure

The first way to calculate the number of atoms is to use vapor pressure from the Steck line data for Rb87. At 25 Celsius, it seems like the vapor pressure is 30e-7 torr. Plug this into PV=nRT (we calculated V = 2.1e-5 m^3, R = 8.314 J/(mol K), and T = 25 + 273.15 = 298.15 K), and we get n ~ 0.000399967 moles so (multiplying by Avogadro's number), the total number of Rb atoms in the entire vapor cell is 2.04051e+12 atoms. We want to consider the number of atoms within the volume of the laser beam (estimated by a cylinder with diameter of the beam size ~1 mm and length of the vapor cell ~ 75 mm) to get the number of atoms that the photons could actually interact with based on the beam size. We get the number of Rb atoms in the laser beam volume as **5.72363e+09 atoms**. This is a few orders of magnitude larger than the number of photons in the cell at a given time (we calculated to be ~10^6).

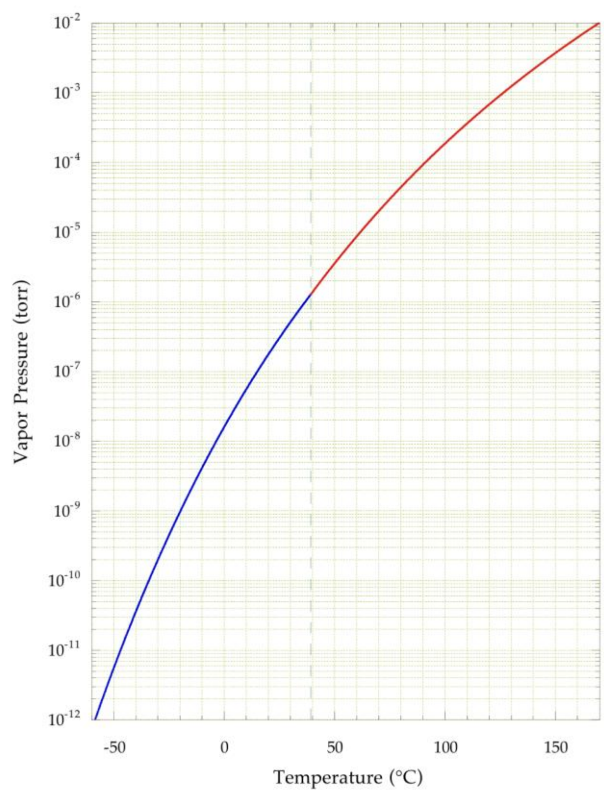


Figure 1: Vapor pressure of ^{87}Rb from the model of Eqs. (1). The vertical line indicates the melting point.

alkali_numbers.dvi (steck.us)

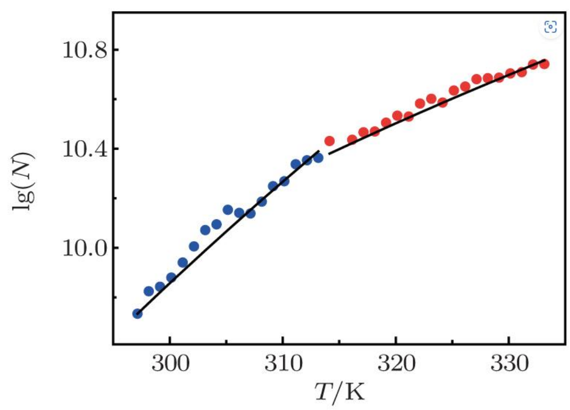
The ideal gas law assumes we aren't operating at low temperatures or high pressures, which I don't think we are. A compressibility chart for rubidium (which would say at which pressures the ideal gas law breaks down), would verify this. We could alternatively use the ideal gas law correcting for alkali metals (Rb being one) in equilibrium gas phase (Thermodynamics of Equilibrium Alkali Plasma. Simple and Accurate Analytical Model for Non-Trivial Case (arxiv.org)). In a nonperfect gas, the pressure will increase much more than expected with an increase in temperature and according to this, the ideal gas law for alkalis is only reasonable along the isotherms T = 856 K and 2547 K. Anyways, I think this is valid for >300 K so it's reasonable for 298 K (the temperature we operated at).

$$p = \rho \frac{k}{\mu} e^{cT}. \qquad (8)$$

Here, b, k and $c = b \ln 10$ are the positive constants. The coefficient k has dimension of (energy)/(amount of substance), whereas dimension of the coefficients b and c is $(\text{temperature})^{-1}$; and, as found, $k = 4.1 \cdot 10^3$ J/mol, $b = 28 \cdot 10^{-5}$ K^{-1} and $c = 64.47 \cdot 10^{-5}$ K^{-1}.

Using the same P from the Steck line data as above, we get: **2.85531e+09 atoms** in the volume of the laser beam.

The third way is to use the measure atomic number density for Rb87, which can be estimated based on the below graph (which was measured using measured fluorescence intensity) as N = 10^9.7 1/cm^3 → **2.95224e08** Rb atoms in the laser beam volume.



Determination of the atomic density of rubidium-87 (iphy.ac.cn)

This website also exists but I don't think these equations apply because the temperature range is much higher than is applicable to our scenario. Thermionic Phenomena Caused by Vapors of Rubidium and Potassium (aps.org)

Anyways, these three ways are within an order of magnitude (the number of atoms in the region that photons will pass through is ~**10^8-9**). This is all a rough estimation but there seems to be more atoms than photons or a similar number of atoms (in the region where you can interact with photons) as the number of photons. So, it's probably not reasonable to say that there's just so many photons that all the atoms are absorbing photons once you reach a certain power: it has to do with the ability of an atom to absorb a photon and the lifetime of the excited state. It would be more useful to figure out which atoms can actually absorb photons (based on absorption cross section). Using Beer's law, you could get N or the number of atoms in a certain volume.

$$\log_{10}(I_0/I) = A = \varepsilon \ell c$$

where

- A is the absorbance
- ε is the molar attenuation coefficient or absorptivity of the attenuating species
- ℓ is the optical path length
- c is the concentration of the attenuating species

- l is the length of the vapor cell
- c is the atomic number density
- epsilon is the absorption cross section.

However, the absorption cross section is dependent on the polarization of light (which would supposedly be p-polarized based on our setup). Atomic number density is dependent on temperature based on the vapor cell so we need to do some other corrections aside from Beer's law. I have to do homework so I will not be pursuing this right now, but it would be cool to do because I don't know how to do that.

Maybe useful:

Calculation of the absorption coefficient for a Doppler-broadened multilevel atom (researchgate.net)

Other resources for determining vapor pressure:

Determination of the vapor pressure of rubidium by optical absorption

Vapor pressure of rubidium between 250 and 298 K determined by combined fluorescence and absorption measurements (optica.org)